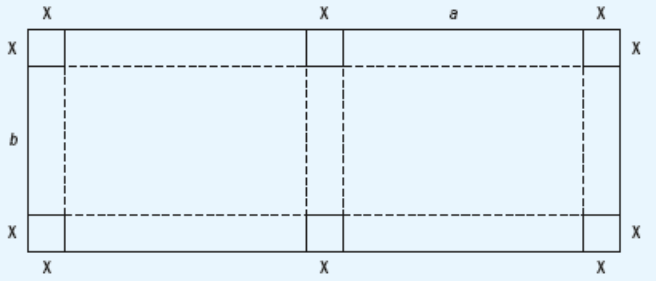
Chapter 4 / **Example 34**

Optimization problems

A piece of cardboard, measuring 100 cm by 200 cm, is to be made into a box by cutting out small squares, each with side length x, as shown in the diagram. a is the length between the squares on the longer side of the cardboard, and b is the length between the squares on the shorter side of the cardboard.

i Find expressions for a and b in terms of x, and state the constraints on the lengths of x, a, and b.

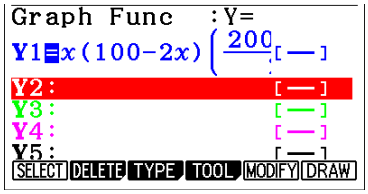
ii Find the value of x (in cm^3) which maximizes the volume of the box, and find the maximum volume of the box. Check your answers graphically.



The problem requires the maximization of the function $V = x(100-2x)\left(\frac{200-3x}{2}\right)$, $x < 50$.

Press MENU 5 GRAPH to display the equation entry screen.

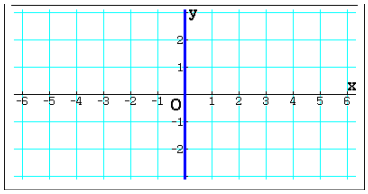
Type $(x+1)(x-3)^3$ and press EXE to enter the equation as Y1.



Press F6 DRAW to display the graph screen.

The GDC now displays the function $\text{Y1} = (x+1)(x-3)^3$

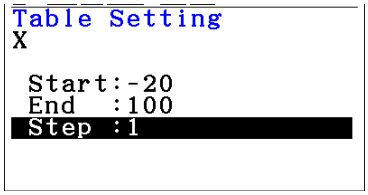
The default axes are $-6.3 \le x \le 6.3$ and $-3.1 \le y \le 3.1$.



To get a better idea of the best window to view the graph in, it is helpful to use a table of values.

Press MENU 7 TABLE. Press F5 SET and change the settings so that the table starts from −20 and ends at 100.

Press EXIT.



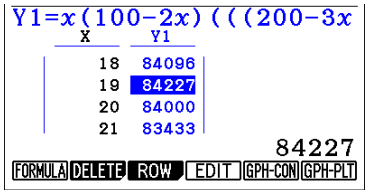
Chapter 4 / **Example 34**

Optimization problems

Press F6 TABLE.

A table of values is displayed.

You can scroll through the table using ▼ and ▲ ▪



Use this information to choose suitable window settings to display the graph.

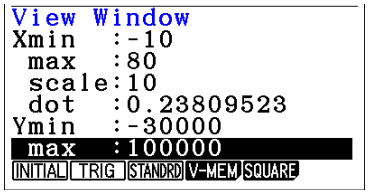
Press MENU 5 GRAPH

Press SHIFT F3 V-WIN.

Set the axes to show $-10 \leq x \leq 80$ with a scale of 10 and $-30\,000 \leq y \leq 100\,000$ with a scale of 10 000.

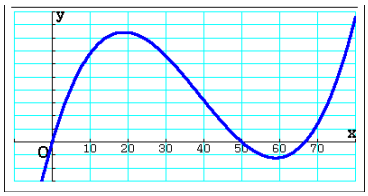
Leave all other items the same.

Press EXIT when you have finished.



Press F6 DRAW to display the graph screen.

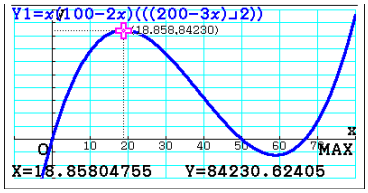
The GDC now displays the curve in a suitable window.



To find the maximum press F5 G-Solv F2 MAX.

Press EXE to display the coordinates.

Press EXIT to leave G-Solv mode and F6 DRAW to display the graph screen again.



The GDC displays the local maximum point at $(18.9, 84\,200)$.

There is also a minimum point which occurs when $x > 50$ which can be rejected.

So the maximum volume is 84 200 cm^3 when x = 18.9 cm.